

Name: _____ Period: _____

14.3 Simplifying Trigonometric Expressions

Simplify the following trig expressions completely:

1. $\tan^2 t (\csc^2 t - 1)$

2. $\cot t (\tan t + \cot t)$

3. $\frac{1 - \cos^2 t}{\sin^2 t}$

4. $\cos t \csc t (\sec^2 t - 1)$

5. $\sec t \tan t \cos t$

6. $\frac{\tan^2 t + 1}{1 + \cot^2 t}$

7. $\frac{\tan^2 t}{1 - \sec^2 t}$

8. $\frac{\sec t \tan t}{\tan^2 t + 1}$

9. $\sin^2 t \cot t \csc t$

10. $\frac{\sin t}{1 - \cos t} + \frac{1 - \cos t}{\sin t}$

11. $\frac{\cos t}{1 + \sin t} + \frac{\cos t}{1 - \sin t}$

12. $\frac{\sin t (1 + \sin t)}{1 - \cos^2 t} - 1$

14.3 Verifying Trigonometric Identities

Verify the following trigonometric identities:

1. $\sec x \cot x = \csc x$

2. $\frac{\tan^2 x}{\sec x} = \sec x - \cos x$

3. $\tan\left(\frac{\pi}{2} - x\right) \sin x = \cos x$

4. $\sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$

5. $\cos x (\csc x + \tan x) = \cot x + \sin x$

6. $\frac{1}{\sin x} - \sin x = \cot x \cos x$

14.3 Simplifying Trigonometric Expressions

Simplify the following trig expressions completely:

$$1. \tan^2 t (\csc^2 t - 1)$$

$$= \frac{\sin^2 t}{\cos^2 t} \cdot \cot^2 t$$

$$= \frac{\sin^2 t}{\cos^2 t} \cdot \frac{\cos^2 t}{\sin^2 t}$$

$$= \boxed{1}$$

$$2. \cot t (\tan t + \cot t)$$

$$\frac{\cos x}{\sin x} \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$\frac{\cos x \cdot \sin x}{\sin x \cdot \cos x} + \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{1 + \cot^2 x}{\csc^2 x}$$

$$3. \frac{1 - \cos^2 t}{\sin^2 t} = \frac{\sin^2 t}{\sin^2 t} = \boxed{1}$$

$$4. \cos t \csc t (\sec^2 t - 1)$$

$$\cos t \left(\frac{1}{\sin t} \right) (\tan^2 t)$$

$$\frac{\cos t}{\sin t} \cdot \frac{\sin^2 t}{\cos^2 t}$$

$$\frac{\sin t}{\cos t} = \boxed{\tan t}$$

$$5. \sec t \tan t \cos t$$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \cdot (\cos x)$$

$$\boxed{\tan x}$$

$$6. \frac{\tan^2 t + 1}{1 + \cot^2 t} = \frac{\sec^2 \theta}{\csc^2 \theta}$$

$$\frac{\frac{1}{\cos^2 \theta}}{\frac{1}{\sin^2 \theta}} = \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} = \boxed{\tan^2 \theta}$$

$$7. \frac{\tan^2 t}{1 - \sec^2 t} = \frac{\tan^2 \theta}{-\tan^2 \theta}$$

$$= \boxed{-1}$$

$$8. \frac{\sec t \tan t}{\tan^2 t + 1} = \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}{\sec^2 x}$$

$$= \frac{\frac{\sin x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\sin x \cdot \cos^2 x}{\cos^2 x \cdot 1}$$

$$= \boxed{\sin x}$$

$$9. \sin^2 t \cot t \csc t$$

$$\sin^2 t \left(\frac{\cos t}{\sin t} \right) \cdot \frac{1}{\sin t}$$

$$\frac{\sin^2 t \cdot \cos t}{\sin^2 t} = \boxed{\cos t}$$

$$10. \frac{\sin t}{(1 + \cos t)(1 - \cos t)} + \frac{1 - \cos t (\sin t)}{\sin t (\sin t)}$$

$$11. \frac{\cos t}{(1 - \sin t)(1 + \sin t)} + \frac{\cos t (1 + \sin t)}{(1 - \sin t)(1 + \sin t)}$$

$$12. \frac{\sin t (1 + \sin t)}{1 - \cos^2 t} - 1$$

$$= \frac{(1 + \cos t) \sin t}{1 - \cos^2 t} + \frac{(1 - \cos t) (\sin t)}{\sin^2 t}$$

$$= \frac{\sin t + \cos t \sin t}{\sin^2 t} + \frac{\sin t - \cos t \sin t}{\sin^2 t}$$

$$\frac{2 \sin t}{\sin^2 t} = \frac{2}{\sin t} = \boxed{2 \csc t}$$

$$\frac{\cos t - \sin t \cos t + \cos t + \sin t \cos t}{1 - \sin^2 t}$$

$$\frac{2 \cos t}{\cos^2 t} = \frac{2}{\cos t} = \boxed{2 \sec t}$$

$$\frac{\sin t (1 + \sin t)}{\sin^2 t} - 1$$

$$\frac{(1 + \sin t)}{\sin t} - 1$$

$$\frac{1 + \sin t}{\sin t} - \frac{\sin t}{\sin t}$$

$$\frac{1 + \sin t - \sin t}{\sin t} = \frac{1}{\sin t} = \boxed{\csc t}$$

14.3 Verifying Trigonometric Identities

Verify the following trigonometric identities:

1. $\sec x \cot x = \csc x$

$$\frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x}$$

$$\frac{1}{\sin x} = \boxed{\csc x}$$

2. $\frac{\tan^2 x}{\sec x} = \sec x - \cos x$

$$\frac{\sec^2 x - 1}{\sec x} = \frac{\sec^2 x}{\sec x} - \frac{1}{\sec x}$$

$$= \sec x - \frac{1}{\sec x}$$

$$= \boxed{\sec x - \cos x}$$

3. $\tan\left(\frac{\pi}{2} - x\right) \sin x = \cos x$

$$\cot x \cdot \sin x$$

$$\frac{\cos x}{\sin x} \cdot \sin x$$

$$\boxed{\cos x}$$

4. $\sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$

$$\sin^2 x (1 - \sin^2 x)$$

$$\sin^2 x (\cos^2 x)$$

$$(1 - \cos^2 x)(\cos^2 x)$$

$$\boxed{\cos^2 x - \cos^4 x}$$

5. $\cos x (\csc x + \tan x) = \cot x + \sin x$

$$\cos x \cdot \csc x + \cos x \cdot \tan x$$

$$\cos x \cdot \left(\frac{1}{\sin x}\right) + \cos x \cdot \left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\cos x}{\sin x} + \sin x$$

$$\boxed{\cot x + \sin x}$$

6. $\frac{1}{\sin x} - \sin x = \cot x \cos x$

$$\frac{1}{\sin x} - \sin x \cdot \left(\frac{\sin x}{\sin x}\right)$$

$$\frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$$= \frac{\cos x \cdot \cos x}{\sin x} = \frac{\cos x}{\sin x} \cdot \cos x$$

$$= \boxed{\cot x \cdot \cos x}$$