

Solving Trigonometric Equations

Solve the equation in the interval $0 \leq x < 2\pi$.

1. $\sqrt{2} \cos x - 1 = 0$

2. $3 \sin x = \sin x + 1$

3. $5 \cos x - \sqrt{3} = 3 \cos x$

4. $4 \sin^2 x - 3 = 0$

5. $3 \tan^2 x - 1 = 0$

6. $1 - 2 \sin^2 x = 0$

7. $3 \tan^3 x - \tan x = 0$

8. $2 \cos x - 4 \cos^2 x = 0$

$$9. \sqrt{2} \cos x \sin x - \cos x = 0$$

$$10. 2 \sin^4 x - \sin^2 x = 0$$

$$11. 2 \sin^2 x + \sin x = 1$$

$$12. \cos^2 x - 4 \cos x + 1 = 0$$

$$13. \sin^2 x - 3 \sin x - 5 = 0$$

$$14. 9 \sin^2 x + 2 = 3$$

Solve the equation in the interval $0 \leq x < 2\pi$.

$$1. \frac{\sqrt{2} \cos x - 1}{+1 \quad +1} = 0$$

$$\frac{\sqrt{2} \cos x - 1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$2. \frac{3 \sin x = \sin x + 1}{-\sin x \quad -\sin x}$$

$$\frac{2 \sin x - 1}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$3. \frac{5 \cos x - \sqrt{3} = 3 \cos x}{-3 \cos x \quad -3 \cos x}$$

$$\frac{2 \cos x - \sqrt{3}}{+ \sqrt{3} \quad + \sqrt{3}} = 0$$

$$\frac{2 \cos x - \sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

$$4. \frac{4 \sin^2 x - 3 = 0}{+3 \quad +3}$$

$$\frac{4 \sin^2 x - 3}{4} = \frac{3}{4}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{3}{4}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$5. \frac{3 \tan^2 x - 1 = 0}{+1 \quad +1}$$

$$\frac{3 \tan^2 x - 1}{3} = \frac{1}{3}$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$6. \frac{1 - 2 \sin^2 x = 0}{-1 \quad -1}$$

$$\frac{-2 \sin^2 x - 1}{-2} = \frac{-1}{-2}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$7. 3 \tan^3 x - \tan x = 0$$

$$\tan x (3 \tan^2 x - 1) = 0$$

$$\tan x = 0$$

$$x = 0, \pi$$

$$3 \tan^2 x - 1 = 0$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$8. 2 \cos x - 4 \cos^2 x = 0$$

$$2 \cos x (1 - 2 \cos x) = 0$$

$$2 \cos x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{1 - 2 \cos x = 0}{-1 \quad -1}$$

$$\frac{-2 \cos x - 1}{-2} = \frac{-1}{-2}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$9. \sqrt{2} \cos x \sin x - \cos x = 0$$

$$\cos x (\sqrt{2} \sin x - 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sqrt{2} \sin x - 1 = 0$$

$$\frac{\sqrt{2} \sin x}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$11. 2 \sin^2 x + \sin x = 1$$

$$\sin x (2 \sin x + 1) = 1$$

$$2 \sin^2 x + \sin x - 1 = 0$$

$$2x^2 + x - 1 = 0$$

$$\sin x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$$

$$\sin x = \frac{-1 \pm \sqrt{9}}{4}$$

$$\sin x = \frac{-1 \pm 3}{4} \rightarrow \begin{cases} \sin x = \frac{1}{2} \\ \sin x = -1 \end{cases}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$13. \sin^2 x - 3 \sin x - 5 = 0$$

$$x^2 - 3x - 5 = 0$$

$$\sin x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$$

$$\sin x = \frac{3 \pm \sqrt{29}}{2}$$

$$x = \sin^{-1}\left(\frac{3 + \sqrt{29}}{2}\right)$$

$$x = \text{N.S.}$$

$$x = \sin^{-1}\left(\frac{3 - \sqrt{29}}{2}\right)$$

$$x = \text{N.S.}$$

$$10. 2 \sin^4 x - \sin^2 x = 0$$

$$\sin^2 x (2 \sin^2 x - 1) = 0$$

$$\sqrt{\sin^2 x} = \sqrt{0}$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$2 \sin^2 x - 1 = 0$$

$$\frac{2 \sin^2 x}{2} = \frac{1}{2}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$12. \cos^2 x - 4 \cos x + 1 = 0$$

$$x^2 - 4x + 1 = 0$$

$$\cos x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$\cos x = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$$x = \cos^{-1}(2 + \sqrt{3}) \quad x = \cos^{-1}(2 - \sqrt{3})$$

$$x = \text{N.S.} \quad x = 1.3$$

$$x = 2\pi - 1.3 = 4.98$$

$$14. 9 \sin^2 x + 2 = 3$$

$$\frac{9 \sin^2 x}{9} = \frac{1}{9}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{9}}$$

$$\sin x = \pm \frac{1}{3}$$

$$x = \sin^{-1}\left(\frac{1}{3}\right)$$

$$x = 0.34$$

$$x = \pi - 0.34$$

$$x = 2.8$$

$$x = \sin^{-1}\left(-\frac{1}{3}\right)$$

$$x = -0.34$$

$$x = 2\pi - 0.34$$

$$x = 5.94$$

$$x = \pi + 0.34$$

$$x = 3.48$$